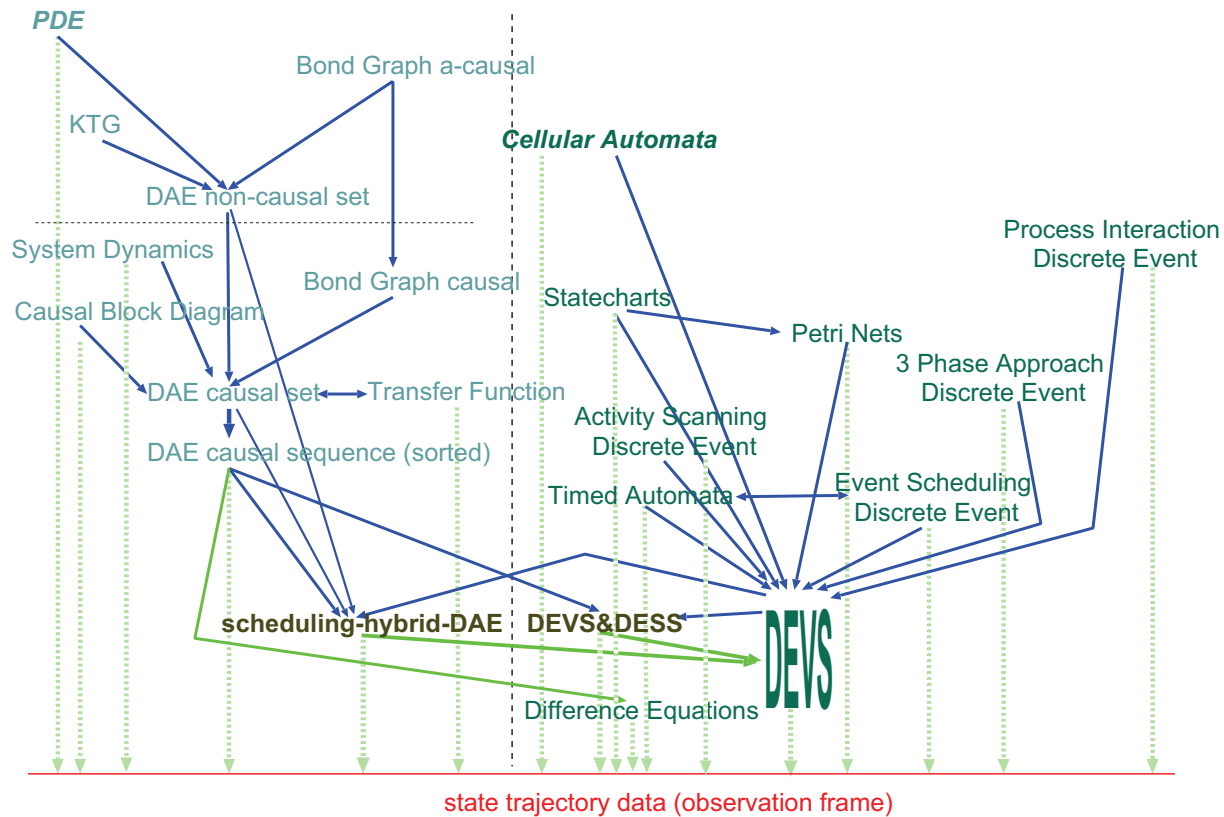


# Discrete Event System specification (DEVS)

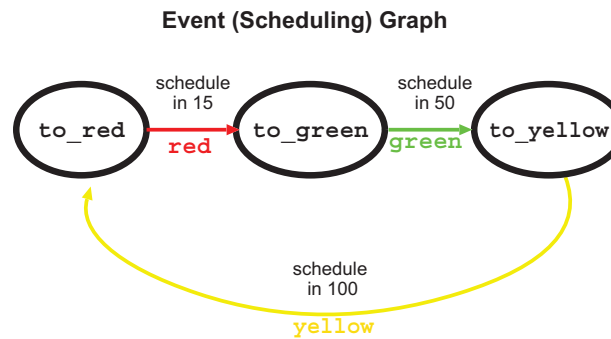
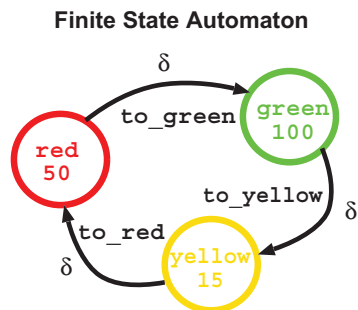
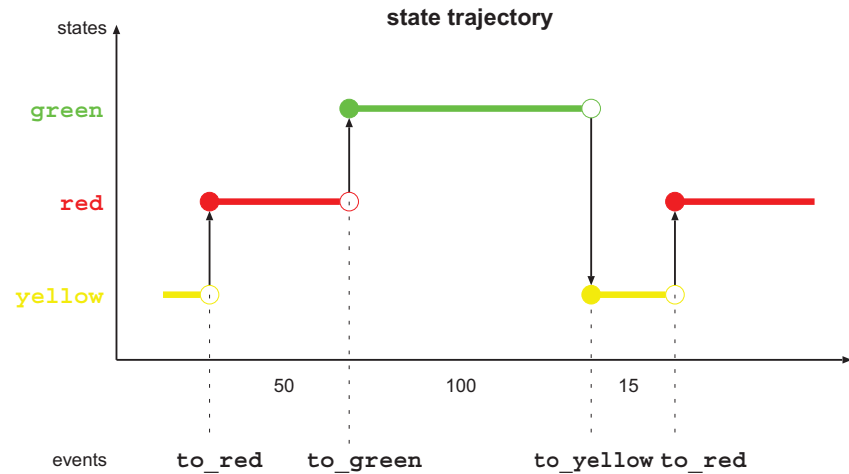
Bernard Zeigler (1976 “Theory of Modelling and Simulation”)

- A formal basis
- for (low-level) representation
- of *all* discrete event modelling formalisms  
(and even others, after approximation)
- and simulator implementations

# DEVS's central place in the Formalism Transformation Graph



# Event Graphs



# DEVS without external events (model)

## Operational Semantics

variables:

time = 0

current\_state = s2

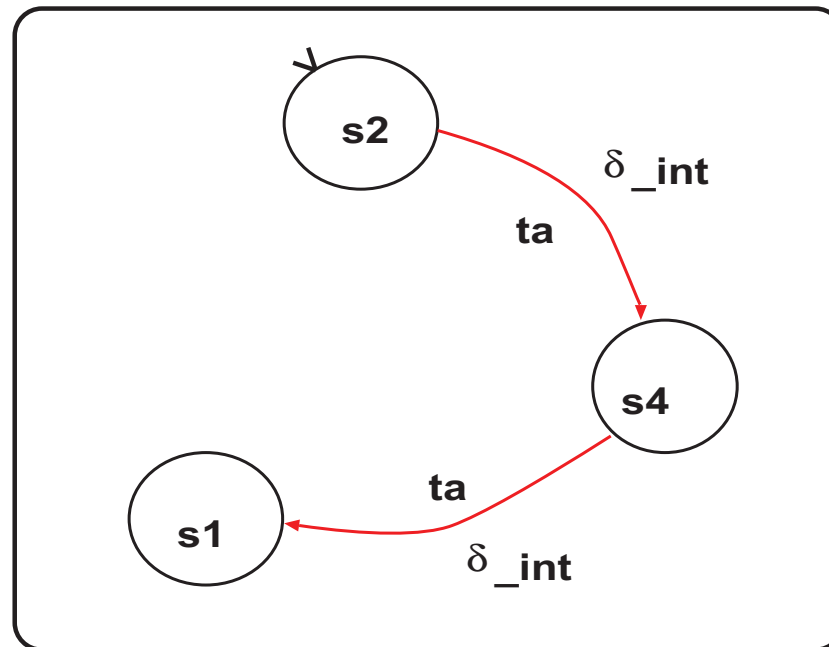
simulator:

while True:

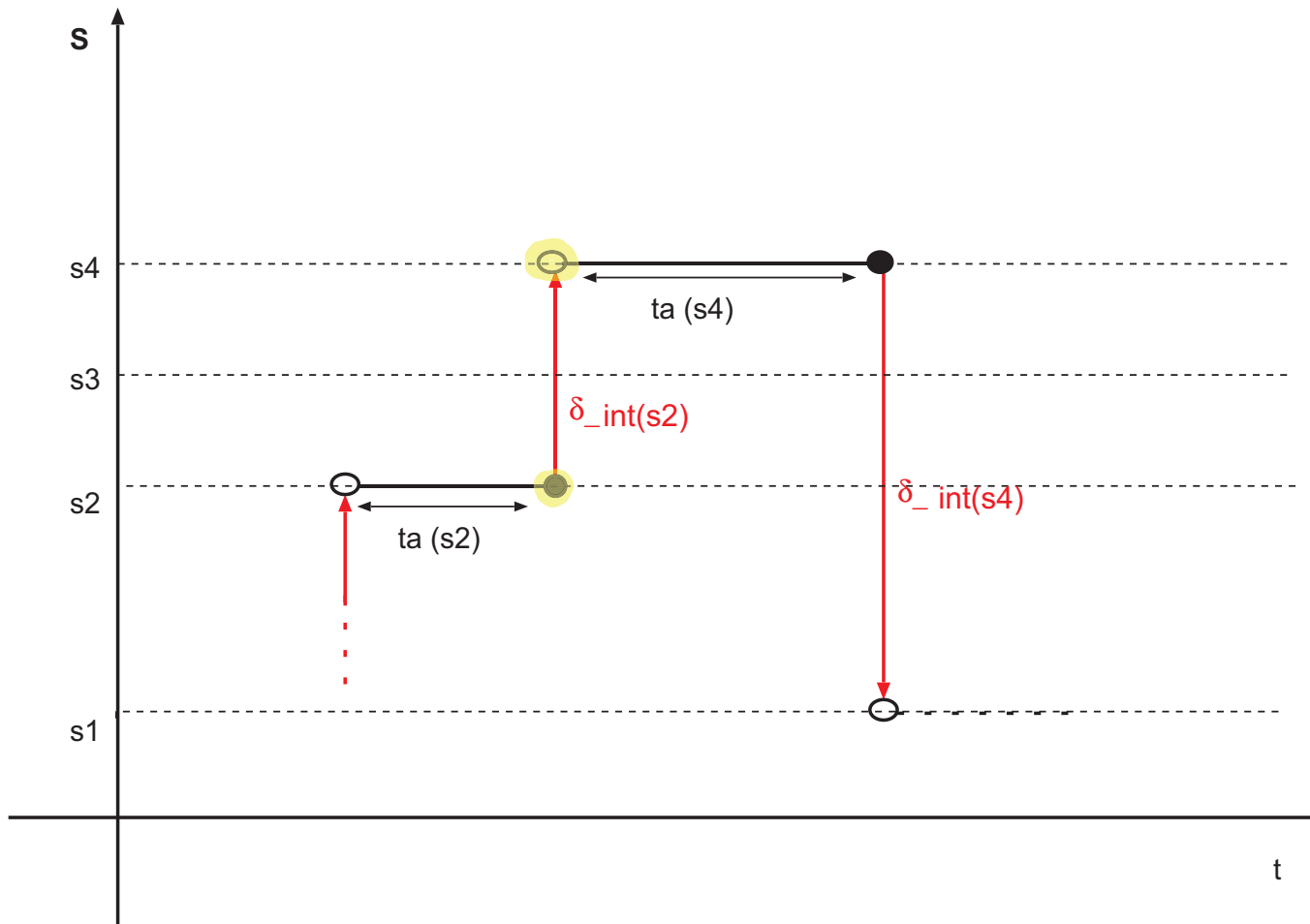
time += ta(current\_state)

current\_state =

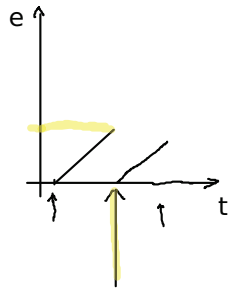
delta\_int(current\_state)



# DEVS without external events (trajectories)



# DEVS with external events (model)



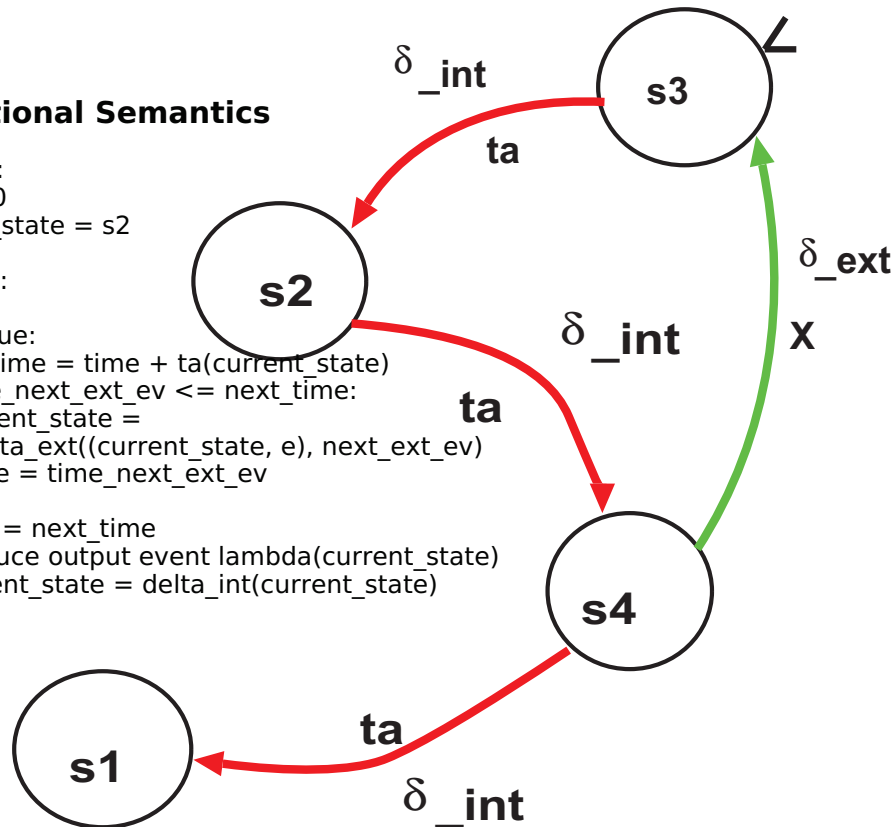
## Operational Semantics

variables:  
 time = 0  
 current\_state = s2

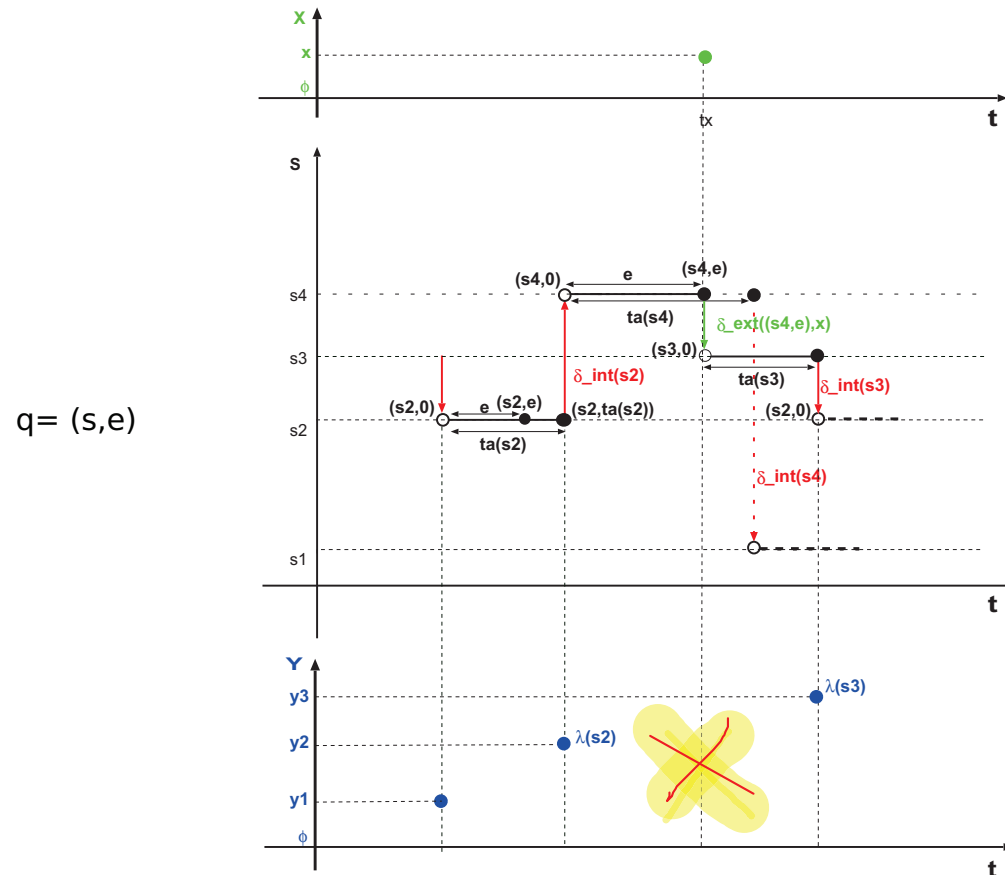
simulator:

```

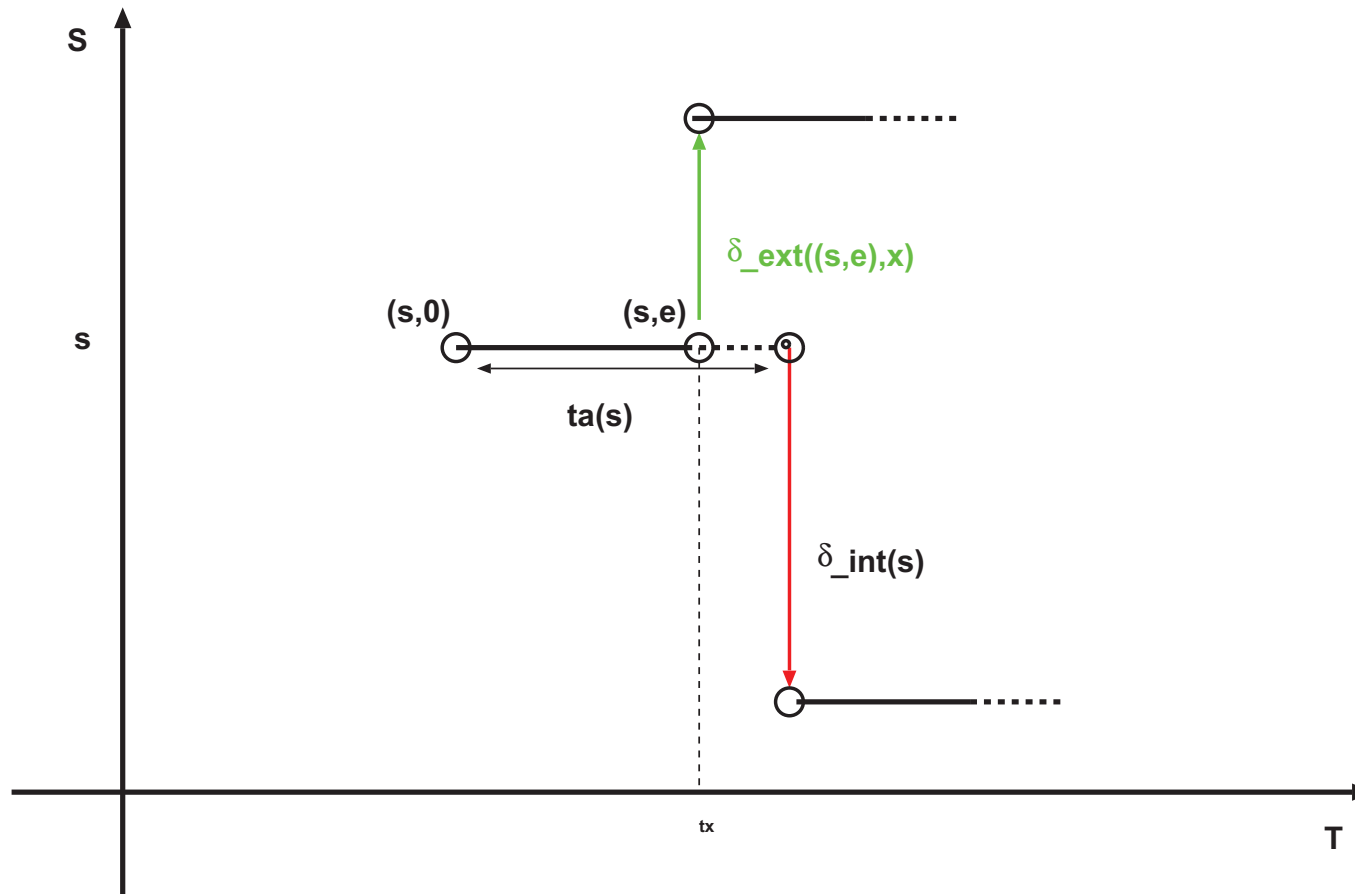
while True:
    next_time = time + ta(current_state)
    if time_next_ext_ev <= next_time:
        current_state =
            delta_ext((current_state, e), next_ext_ev)
        time = time_next_ext_ev
    else:
        time = next_time
        produce output event lambda(current_state)
        current_state = delta_int(current_state)
    
```



# DEVS with external events (trajectories)



# DEVS essence

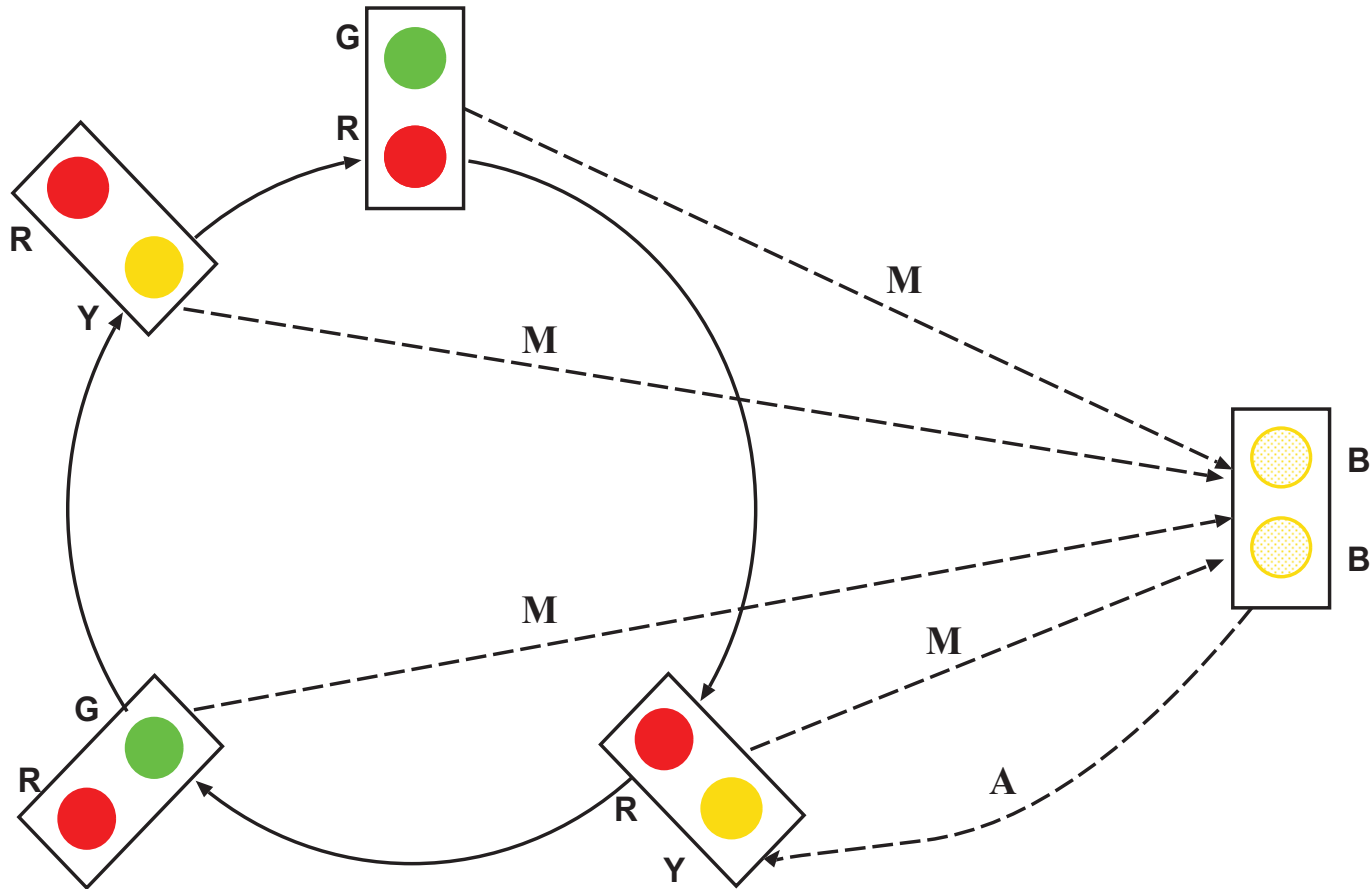




$$DEVS = \langle X, S, Y, \delta_{int}, \delta_{ext}, \lambda, ta \rangle$$

$T = \mathbb{R}$	time base
$X$	input set
$\omega : T \rightarrow X \cup \{\phi\}$	input segment
$S$	state set
$Y$	output set
$\delta_{int} : S \rightarrow S$	internal transition function
$ta : S \rightarrow \mathbb{R}_{0, \infty}^+$	time advance function
$Q = \{(s, e)   s \in S, 0 \leq e \leq ta(s)\}$	total state, $e$ is elapsed time
$\delta_{ext} : Q \times X \rightarrow S$	external transition function
$\lambda : S \rightarrow Y$	output function

# Traffic Lights



$$\text{trafficDEVs} = \langle X, S, Y, \delta_{int}, \delta_{ext}, \lambda, ta \rangle$$

$$T = \mathbb{R}$$

$$X = \{M, A\}$$

$$\omega : T \rightarrow X \cup \{\phi\}$$

$$S = \{RG, RY, GR, YR, BB\}$$

$$\delta_{int}(RG) = RY; \delta_{int}(RY) = GR$$

$$\delta_{int}(GR) = YR; \delta_{int}(YR) = RG$$

$$ta(RG) = 60s; ta(RY) = 10s$$

$$ta(GR) = 50s; ta(YR) = 10s$$

$$ta(BB) = +\infty$$

$trafficDEVS = \langle X, S, Y, \delta_{int}, \delta_{ext}, \lambda, ta \rangle$

$$\delta_{ext}((RG, e), M) = BB$$

$$\delta_{ext}((RY, e), M) = BB$$

$$\delta_{ext}((GR, e), M) = BB$$

$$\delta_{ext}((YR, e), M) = BB$$

$$\delta_{ext}((BB, e), A) = RY$$

$$Y = \{GREY, YELLOW, BLINK\}$$

$$\lambda(RG) = \lambda(RY) = \lambda(GR) = GREY$$

$$\lambda(YR) = YELLOW$$

$$\lambda(BB) = BLINK$$

# Coupled DEVS

$$\text{coupledDEVS} \equiv \langle X_{self}, Y_{self}, D, \{M_i\}, \{I_i\}, \{Z_{i,j}\}, select \rangle \\ \{M_i | i \in D\}.$$

$$M_i = \langle S_i, ta_i, \delta_{int,i}, X_i, \delta_{ext,i}, Y_i, \lambda_i \rangle, \forall i \in D.$$

$$\{I_i | i \in D \cup \{self\}\}.$$

$$\forall i \in D \cup \{self\} : I_i \subseteq D \cup \{self\}.$$

$$\forall i \in D \cup \{self\} : i \notin I_i.$$

$I_i$  are the influence sets describing the connection topology

## $Z_{i,j}$ output-to-input translation

$$\{Z_{i,j} | i \in D \cup \{self\}, j \in I_i\},$$

$$Z_{self,j} : X_{self} \rightarrow X_j \quad , \forall j \in D,$$

$$Z_{i,self} : Y_i \rightarrow Y_{self} \quad , \forall i \in D,$$

$$Z_{i,j} : Y_i \rightarrow X_j \quad , \forall i, j \in D.$$

Together,  $I_i$  and  $Z_{i,j}$  completely specify the coupling (structure and behaviour)

# Tie-breaking among simultaneous events

$$\textit{select} : 2^D \rightarrow D$$

Choose a unique component from any non-empty subset  $E$  of  $D$ :

$$\textit{select}(E) \in E.$$

$E$  corresponds to the set of all components having a state transition simultaneously (*collisions*).

# Closure under coupling

From the coupled DEVS

$$\langle X_{self}, Y_{self}, D, \{M_i\}, \{I_i\}, \{Z_{i,j}\}, select \rangle,$$

with all components  $M_i$  atomic DEVS models

$$M_i = \langle S_i, ta_i, \delta_{int,i}, X_i, \delta_{ext,i}, Y_i, \lambda_i \rangle, \forall i \in D$$

the atomic DEVS

$$\langle S, ta, \delta_{int}, X, \delta_{ext}, Y, \lambda \rangle$$

is constructed.



## Closure: state and time-advance

$$S = \times_{i \in D} Q_i,$$

where

$$Q_i = \{(s_i, e_i) | s \in S_i, 0 \leq e_i \leq ta_i(s_i)\}, \forall i \in D.$$

$$ta : S \rightarrow \mathbb{R}_{0, +\infty}^+$$

Select the *most imminent* event time, = smallest time *remaining* until internal transition, of all the components

$$ta(s) = \min\{\sigma_i = ta_i(s_i) - e_i | i \in D\}.$$

# Dealing with simultaneous events

Imminent components:

$$IMM(s) = \{i \in D \mid \sigma_i = ta(s)\}.$$

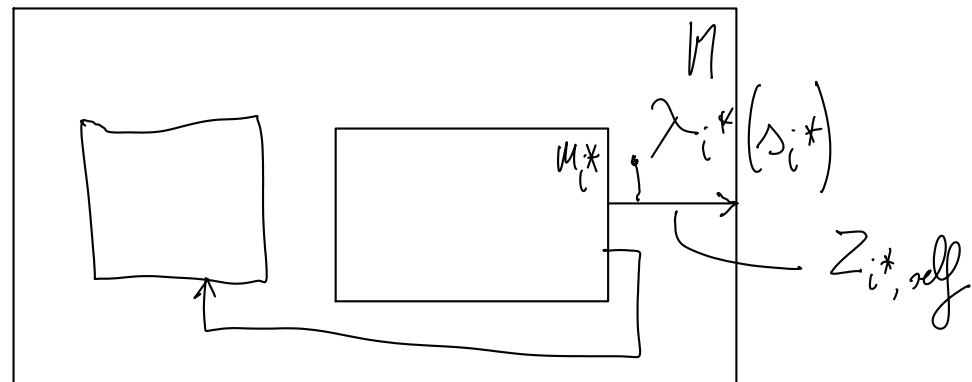
*select one* component  $i^*$  of the coupled model

$$\begin{array}{lcl} \textit{select} & : & 2^D \quad \rightarrow \quad D \\ & & IMM(s) \quad \rightarrow \quad i^* \end{array}$$

# Output (at internal transition time)

$$\lambda(s) = \begin{cases} Z_{i^*,self}(\lambda_{i^*}(s_{i^*})) & ,\text{if } self \in I_{i^*}, \\ \phi & ,\text{if } self \notin I_{i^*}. \end{cases}$$

Conceptually, the non-event  $\phi$  is generated if  $i^*$  is not connected to the output of the coupled model.



# Internal transition function

$$\left( \overbrace{(\lambda_1, e_1), (\lambda_2, e_2), \dots, (\lambda_n, e_n)}^{\lambda} \right)$$

$$\delta_{int}(s) = (\dots, (s'_j, e'_j), \dots), \text{ where}$$

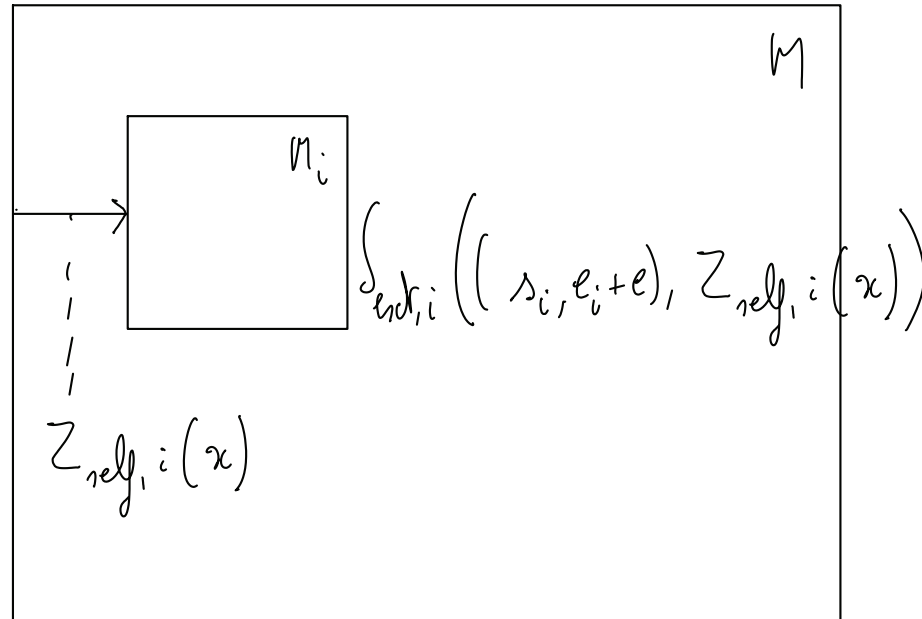
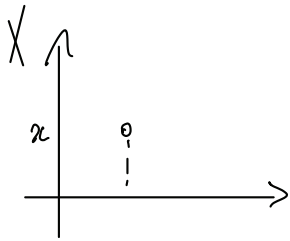
$$\begin{aligned} (s'_j, e'_j) &= (\delta_{int,j}(s_j), 0) && \text{, for } j = i^*, \\ &= (\delta_{ext,j}(s_j, e_j + ta(s), Z_{i^*,j}(\lambda_{i^*}(s_{i^*}))), 0) && \text{, for } j \in I_{i^*} \\ &&& \text{(and } Z_{i^*,j}(\lambda_{i^*}(s_{i^*})) \neq \epsilon) \\ &= (s_j, e_j + ta(s)) && \text{, otherwise.} \end{aligned}$$

# External transition function

$\delta_{ext}(s, e, x) = (\dots, (s'_i, e'_i), \dots)$ , where

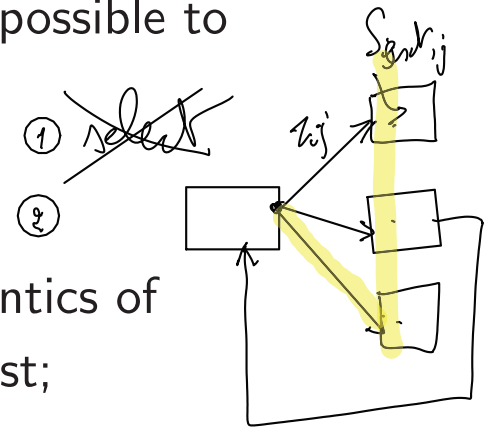
$$\begin{aligned} (s'_i, e'_i) &= (\delta_{ext,i}(s_i, \underline{e_j + e}, Z_{self,i}(x)), 0) \quad , \text{for } i \in I_{self}, \\ &= (s_i, \underline{e_i + e}) \quad , \text{otherwise.} \end{aligned}$$

$$i \in I_{\text{rel}}$$



# DEVS limitations

- a conflict due to simultaneous internal and external events is resolved by ignoring the internal event. It should be possible to explicitly specify behaviour in case of conflicts;
- there is limited potential for parallel implementation;
- the *select* function is an artificial legacy of the semantics of traditional sequential simulators based on an event list;
- it is not possible to explicitly describe variable structure.



DS-DEVS

Some of these are resolved in *parallel DEVS*

# DEVS Solver

- Iterative simulation of DEVS model
- Possibly distributed implementation



message $m$	simulator	coordinator
$(*, from, t)$	<p>simulator correct only if <math>t = t_N</math></p> <p><math>y \leftarrow \lambda(s)</math></p> <p><b>if</b> <math>y \neq \phi</math> :</p> <p>    <b>send</b> <math>(\lambda(s), self, t)</math> to parent</p> <p><math>s \leftarrow \delta_{int}(s)</math></p> <p><math>t_L \leftarrow t</math></p> <p><math>t_N \leftarrow t_L + ta(s)</math></p> <p><b>send</b> <math>(done, self, t_N)</math> to parent</p>	<p><b>send</b> <math>(*, self, t)</math> to <math>i^*</math>, where</p> <p><math>i^* = select(imm\_children)</math></p> <p><math>imm\_children = \{i \in D \mid M_i.t_N = t\}</math></p> <p><math>active\_children \leftarrow active\_children \cup \{i^*\}</math></p>

---

message $m$	simulator	coordinator
$(x, from, t)$	<p>simulator correct only if <math>t_L \leq t \leq t_N</math> (ignore <math>\delta_{int}</math> to resolve a <math>t = t_N</math> conflict)</p> <p><math>e \leftarrow t - t_L</math></p> <p><math>s \leftarrow \delta_{ext}(s, e, x)</math></p> <p><math>t_L \leftarrow t</math></p> <p><math>t_N \leftarrow t_L + ta(s)</math></p> <p><b>send</b> <math>(done, self, t_N)</math> to parent</p>	<p><math>\forall i \in I_{self} :</math></p> <p><b>send</b> <math>(Z_{self,i}(x), self, t)</math> to <math>i</math></p> <p><math>active\_children \leftarrow active\_children \cup \{i\}</math></p>

---

---

message $m$	simulator	coordinator
$(y, from, t)$		$\forall i \in I_{from} \setminus \{self\} :$ <b>send</b> $(Z_{from,i}(y), from, t)$ to $i$ $active\_children \leftarrow active\_children \cup \{i\}$ <b>if</b> $self \in I_{from} :$ <b>send</b> $(Z_{from,self}(y), self, t)$ to parent
$(done, from, t)$		$active\_children \leftarrow active\_children \setminus \{from\}$ <b>if</b> $active\_children = \emptyset :$ $t_L \leftarrow t$ $t_N \leftarrow \min\{M_i.t_N   i \in D\}$ <b>send</b> $(done, self, t_N)$ to parent

---

# DEVS simulator main loop

---

$t \leftarrow t_N$  of topmost coordinator

**repeat until**  $t \geq t_{end}$  (or some other termination condition)

**send**  $(*, main, t)$  to topmost coupled model  $top$

**wait** for  $(done, top, t_N)$

$t \leftarrow t_N$

---